

Confidence Intervals for Means

Lecture 35
Section 10.4

Robb T. Koether

Hampden-Sydney College

Wed, Mar 28, 2012

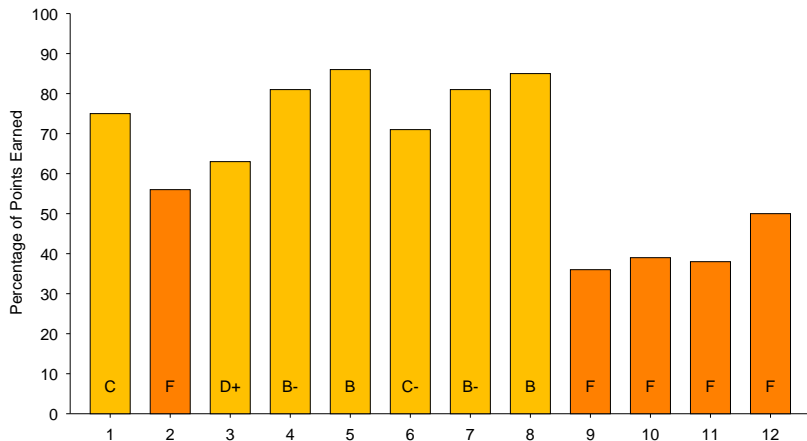
Outline

- 1 Review of Test 2
- 2 Introduction
- 3 Confidence Intervals for the Mean
- 4 Choosing the Statistic
- 5 Examples
 - Using z
 - Using t
- 6 Confidence Intervals on the TI-83
 - Using `ZInterval`
 - Using `TInterval`
- 7 Assignment

Outline

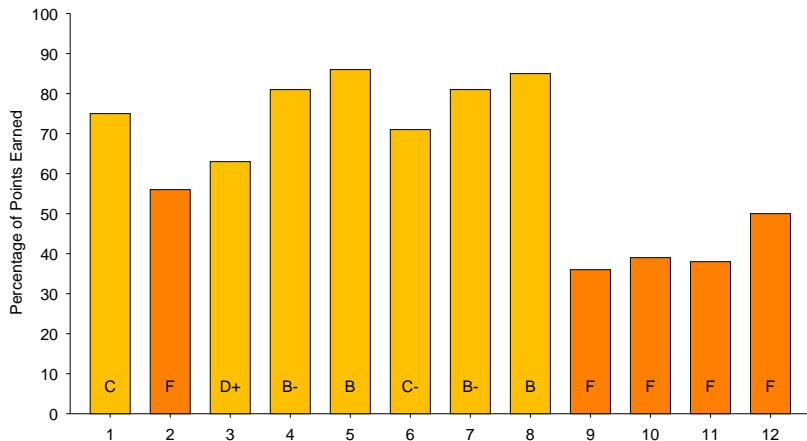
- 1 Review of Test 2
- 2 Introduction
- 3 Confidence Intervals for the Mean
- 4 Choosing the Statistic
- 5 Examples
 - Using z
 - Using t
- 6 Confidence Intervals on the TI-83
 - Using `ZInterval`
 - Using `TInterval`
- 7 Assignment

An Analysis of Test 2



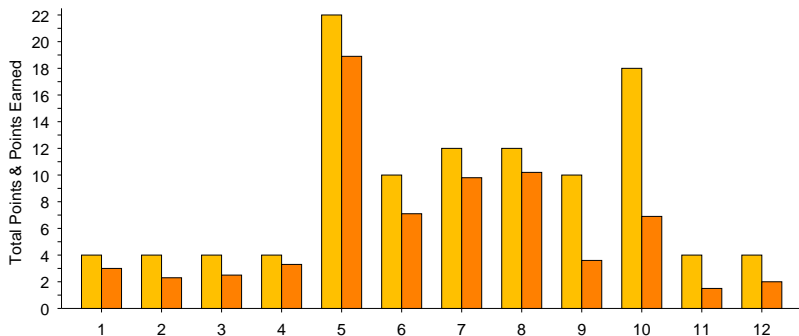
The difficulty of the questions

An Analysis of Test 2



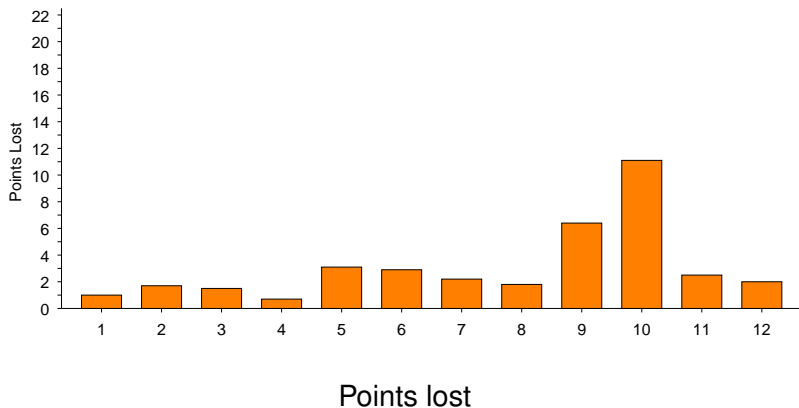
Or the preparedness of the students?

An Analysis of Test 2

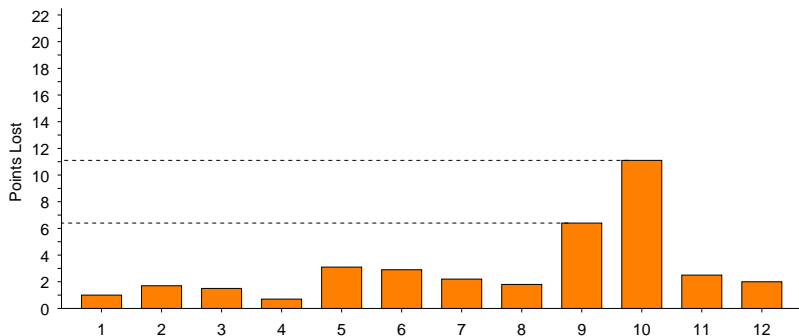


Points earned vs. points possible

An Analysis of Test 2



An Analysis of Test 2



17 points lost on Problems 9 and 10

Outline

- 1 Review of Test 2
- 2 Introduction**
- 3 Confidence Intervals for the Mean
- 4 Choosing the Statistic
- 5 Examples
 - Using z
 - Using t
- 6 Confidence Intervals on the TI-83
 - Using `ZInterval`
 - Using `TInterval`
- 7 Assignment

- In chapter 9 we computed confidence intervals for p :

$$\hat{p} \pm z \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}.$$

- In chapter 9 we computed confidence intervals for p :

$$\hat{p} \pm z \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}.$$

- Now we will compute confidence intervals for μ .

- In chapter 9 we computed confidence intervals for p :

$$\hat{p} \pm z \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}.$$

- Now we will compute confidence intervals for μ .
- The underlying theory is exactly the same.

- In chapter 9 we computed confidence intervals for p :

$$\hat{p} \pm z \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}.$$

- Now we will compute confidence intervals for μ .
- The underlying theory is exactly the same.
- Therefore, the results follow exactly the same pattern.

Outline

- 1 Review of Test 2
- 2 Introduction
- 3 Confidence Intervals for the Mean**
- 4 Choosing the Statistic
- 5 Examples
 - Using z
 - Using t
- 6 Confidence Intervals on the TI-83
 - Using `ZInterval`
 - Using `TInterval`
- 7 Assignment

Confidence Intervals

- The point estimate for μ is \bar{x} .
- The standard deviation of the point estimator is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.
- The sampling distribution of \bar{x} is $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$.

Confidence Intervals

- The confidence interval for the mean will be

$$\bar{x} \pm z \left(\frac{\sigma}{\sqrt{n}} \right),$$

or

$$\bar{x} \pm t \left(\frac{s}{\sqrt{n}} \right),$$

depending on the circumstances.

Outline

- 1 Review of Test 2
- 2 Introduction
- 3 Confidence Intervals for the Mean
- 4 Choosing the Statistic**
- 5 Examples
 - Using z
 - Using t
- 6 Confidence Intervals on the TI-83
 - Using `ZInterval`
 - Using `TInterval`
- 7 Assignment

Whether to Use z or t

Which statistic?

- If the sample size is **large**, then it **does not matter** whether we use z or t .
- However, the best rule to follow is to use z when σ is known and use t when σ is unknown.

Whether to Use z or t

Which statistic?

- If the sample size is **small**, then it **does matter** and it is necessary to know that the population is normal.
- Use z if σ is known and use t if σ is unknown.

Summary

Std. Dev.	Population	n	Test
σ	Normal	< 30	Z-Test
σ	Any	≥ 30	Z-Test
s	Normal	< 30	T-Test
s	Any	≥ 30	T-Test

Outline

- 1 Review of Test 2
- 2 Introduction
- 3 Confidence Intervals for the Mean
- 4 Choosing the Statistic
- 5 Examples**
 - Using z
 - Using t
- 6 Confidence Intervals on the TI-83
 - Using `ZInterval`
 - Using `TInterval`
- 7 Assignment

Outline

- 1 Review of Test 2
- 2 Introduction
- 3 Confidence Intervals for the Mean
- 4 Choosing the Statistic
- 5 Examples**
 - Using z
 - Using t
- 6 Confidence Intervals on the TI-83
 - Using `ZInterval`
 - Using `TInterval`
- 7 Assignment

Example (Confidence Interval Using z)

- A sample of 40 employees at a company was asked how long their commute to work was.
- They reported an average time of 35 minutes.
- The standard deviation is assumed to be $\sigma = 18$ minutes.
- Construct a 95% confidence interval for the true mean commute time for the employees of this company.

Example (Confidence Interval Using z)

- Do we know σ ?

Example (Confidence Interval Using z)

- Do we know σ ?
- Yes.

Example (Confidence Interval Using z)

- Do we know σ ?
- Yes.
- Is the sample size at least 30?

Example (Confidence Interval Using z)

- Do we know σ ?
- Yes.
- Is the sample size at least 30?
- Yes.

Confidence Intervals Using z

Example (Confidence Interval Using z)

- Do we know σ ?
- Yes.
- Is the sample size at least 30?
- Yes.
- Therefore, use z .

Confidence Intervals Using z

Example (Confidence Interval Using z)

- We use $z = 1.960$ for a 95% confidence interval.
- The confidence interval is

$$\begin{aligned}\bar{x} \pm z \frac{\sigma}{\sqrt{n}} &= 35 \pm 1.960 \left(\frac{18}{\sqrt{40}} \right) \\ &= 35 \pm 5.578.\end{aligned}$$

Outline

- 1 Review of Test 2
- 2 Introduction
- 3 Confidence Intervals for the Mean
- 4 Choosing the Statistic
- 5 Examples**
 - Using z
 - **Using t**
- 6 Confidence Intervals on the TI-83
 - Using `ZInterval`
 - Using `TInterval`
- 7 Assignment

Confidence Intervals Using t

Example (Confidence Interval Using t)

- Now suppose that the company surveyed only 10 employees and obtained the following commute times:

48	15	36	14	29
36	25	56	66	28

- Construct a 95% confidence interval for the mean commute time.

Confidence Intervals Using t

Example (Confidence Interval Using t)

- Do we know σ ?

Confidence Intervals Using t

Example (Confidence Interval Using t)

- Do we know σ ?
- No.

Confidence Intervals Using t

Example (Confidence Interval Using t)

- Do we know σ ?
- No.
- Is the sample size at least 30?

Confidence Intervals Using t

Example (Confidence Interval Using t)

- Do we know σ ?
- No.
- Is the sample size at least 30?
- No.

Confidence Intervals Using t

Example (Confidence Interval Using t)

- Do we know σ ?
- No.
- Is the sample size at least 30?
- No.
- Is the population normal?

Confidence Intervals Using t

Example (Confidence Interval Using t)

- Do we know σ ?
- No.
- Is the sample size at least 30?
- No.
- Is the population normal?
- A QQ plot shows that it is approximately normal.

Confidence Intervals Using t

Example (Confidence Interval Using t)

- Do we know σ ?
- No.
- Is the sample size at least 30?
- No.
- Is the population normal?
- A QQ plot shows that it is approximately normal.
- Therefore, use t .

Confidence Intervals Using t

Example (Confidence Interval Using t)

- Do we know σ ?
- No.
- Is the sample size at least 30?
- No.
- Is the population normal?
- A QQ plot shows that it is approximately normal.
- Therefore, use t .
- (We must use t .)

Confidence Intervals Using t

Example (Confidence Interval Using t)

- The confidence interval is given by

$$\bar{x} \pm t \left(\frac{s}{\sqrt{n}} \right).$$

Confidence Intervals Using t

Example (Confidence Interval Using t)

- The confidence interval is given by

$$\bar{x} \pm t \left(\frac{s}{\sqrt{n}} \right).$$

- $\bar{x} = 35.3$.

Confidence Intervals Using t

Example (Confidence Interval Using t)

- The confidence interval is given by

$$\bar{x} \pm t \left(\frac{s}{\sqrt{n}} \right).$$

- $\bar{x} = 35.3$.
- $s = 16.99$.

Confidence Intervals Using t

Example (Confidence Interval Using t)

- The confidence interval is given by

$$\bar{x} \pm t \left(\frac{s}{\sqrt{n}} \right).$$

- $\bar{x} = 35.3$.
- $s = 16.99$.
- $n = 10$.

Confidence Intervals Using t

Example (Confidence Interval Using t)

- The confidence interval is given by

$$\bar{x} \pm t \left(\frac{s}{\sqrt{n}} \right).$$

- $\bar{x} = 35.3$.
- $s = 16.99$.
- $n = 10$.
- But what is the value of t ?

Confidence Intervals Using t

Example (Confidence Interval Using t)

- The confidence interval is given by

$$\bar{x} \pm t \left(\frac{s}{\sqrt{n}} \right).$$

- $\bar{x} = 35.3$.
- $s = 16.99$.
- $n = 10$.
- But what is the value of t ?
- It is that value that cuts off the upper 2.5% (or the lower 2.5%).

Confidence Intervals Using t

Example (Confidence Interval Using t)

- The confidence interval is given by

$$\bar{x} \pm t \left(\frac{s}{\sqrt{n}} \right).$$

- $\bar{x} = 35.3$.
- $s = 16.99$.
- $n = 10$.
- But what is the value of t ?
- It is that value that cuts off the upper 2.5% (or the lower 2.5%).
- So, what value is that?

The invT Function

- You may have the invT function on your TI-84.
- If so, then you can compute

$$\text{invT}(.025, 9) = -2.262$$

- Or you can compute

$$\text{invT}(.975, 9) = 2.262$$

Confidence Intervals Using t

Example (Confidence Interval Using t)

- Lacking the `invT` function, you may use the t -tables (Appendix, p. A-7).

The t -Tables

Table IV Upper Percentiles of t -distributions

Area to the Right								
df	0.40	0.30	0.20	0.10	0.05	0.025	0.01	0.005
1	0.325	0.727	1.376	3.078	6.314	12.706	31.821	63.657
2	0.289	0.617	1.061	1.885	2.920	4.303	6.965	9.925
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660
120	0.254	0.526	0.845	1.289	1.658	1.980	2.358	2.617
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576

The t -Tables

Table IV Upper Percentiles of t -distributions

Area to the Right								
df	0.40	0.30	0.20	0.10	0.05	0.025	0.01	0.005
1	0.325	0.727	1.376	3.078	6.314	12.706	31.821	63.657
2	0.289	0.617	1.061	1.885	2.920	4.303	6.965	9.925
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660
120	0.254	0.526	0.845	1.289	1.658	1.980	2.358	2.617
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576

Confidence Intervals Using t

Example (Confidence Interval Using t)

- The coefficient is $t = 2.000$.
- The confidence interval is

$$\begin{aligned}\bar{x} \pm t \left(\frac{s}{\sqrt{n}} \right) &= 35.3 \pm 2.262 \left(\frac{16.99}{\sqrt{10}} \right) \\ &= 35.3 \pm 12.153.\end{aligned}$$

Outline

- 1 Review of Test 2
- 2 Introduction
- 3 Confidence Intervals for the Mean
- 4 Choosing the Statistic
- 5 Examples
 - Using z
 - Using t
- 6 Confidence Intervals on the TI-83**
 - Using `ZInterval`
 - Using `TInterval`
- 7 Assignment

Outline

- 1 Review of Test 2
- 2 Introduction
- 3 Confidence Intervals for the Mean
- 4 Choosing the Statistic
- 5 Examples
 - Using z
 - Using t
- 6 Confidence Intervals on the TI-83**
 - Using $Z_{Interval}$**
 - Using $T_{Interval}$
- 7 Assignment

TI-83 Confidence Intervals

Using `ZInterval`

TI-83 Confidence Intervals for μ (`ZInterval`)

- Press `STAT`.
- Select `TESTS`.
- Select `ZInterval`. A window appears requesting information.
- Select `Data` or `Stats`.

TI-83 Confidence Intervals (Stats Option)

Using ZInterval

TI-83 Confidence Intervals for μ (ZInterval)

- Stats option:
 - Enter σ .
 - Enter \bar{x} .
 - Enter n .
 - Enter the level of confidence.
 - Select `Calculate` and press `ENTER`.

TI-83 Confidence Intervals (Stats Option)

Using `ZInterval`

TI-83 Confidence Intervals for μ (`ZInterval`)

- `Data` option:
 - Enter σ .
 - Enter the list containing the data.
 - `Freq` should be 1 unless the frequencies are stored in a separate list.
 - Enter the level of confidence.
 - Select `Calculate` and press `ENTER`.

TI-83 Confidence Intervals

Using `ZInterval`

TI-83 Confidence Intervals for μ (`ZInterval`)

- A window appears containing
 - The title `ZInterval`.
 - The confidence interval in interval notation.
 - The sample mean.
 - The sample size.

TI-83 Confidence Intervals

Using `ZInterval`

Example (TI-83 `ZInterval`)

- Re-work the first example on the TI-83 using `ZInterval`.

Outline

- 1 Review of Test 2
- 2 Introduction
- 3 Confidence Intervals for the Mean
- 4 Choosing the Statistic
- 5 Examples
 - Using z
 - Using t
- 6 Confidence Intervals on the TI-83**
 - Using `ZInterval`
 - Using `TInterval`**
- 7 Assignment

TI-83 Confidence Intervals

Using TInterval

TI-83 Confidence Intervals for μ (TInterval)

- Press STAT.
- Select TESTS.
- Select TInterval. A window appears requesting information.
- Select Data or Stats.

TI-83 Confidence Intervals

Using `TInterval`

TI-83 Confidence Intervals for μ (`TInterval`)

- `Stats` option:
 - Enter \bar{x} .
 - Enter s .
 - Enter n .
 - Enter the level of confidence.
 - Select `Calculate` and press `ENTER`.

TI-83 Confidence Intervals

Using `TInterval`

TI-83 Confidence Intervals for μ (`TInterval`)

- `Data` option:
 - Enter the list containing the data.
 - `Freq` should be 1 unless the frequencies are stored in a separate list.
 - Enter the level of confidence.
 - Select `Calculate` and press `ENTER`.

TI-83 Confidence Intervals

Using TInterval

TI-83 Confidence Intervals for μ (TInterval)

- A window appears containing
 - The title TInterval.
 - The confidence interval in interval notation.
 - The sample mean.
 - The sample standard deviation.
 - The sample size.

TI-83 Confidence Intervals

Using `TInterval`

Example (TI-83 `ZInterval`)

- Re-work the second example on the TI-83 using `TInterval`.

Outline

- 1 Review of Test 2
- 2 Introduction
- 3 Confidence Intervals for the Mean
- 4 Choosing the Statistic
- 5 Examples
 - Using z
 - Using t
- 6 Confidence Intervals on the TI-83
 - Using `ZInterval`
 - Using `TInterval`
- 7 Assignment

Assignment

Homework

- Read Section 10.4, pages 639 - 647.
- Let's Do It! 10.7, 10.8.
- Exercises 19, 21 - 23, 27 - 30, page 647.
- Review Exercises 36 - 40, 42, 43, 45, 46, 49, page 658.